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# NEW RESEARCHES ON MAGNETIZATION BY ROTATION AND THE GYROMAGNETIC RATIOS OF FERROMAGNETIC SUBSTANCES

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1. Introduction. Since 1914, when the first gyromagnetic effect to be observed was discovered,[1] by a method of electromagnetic induction, many researches have been made on the gyromagnetic ratios of magnetic substances. In the case of ferromagnetic substances, successful measurements of this ratio (i. e. the ratio of the angular momentum of the magnetic element to its magnetic moment) have been made by investigating two different, but closely related effects, viz: (1) *magnetization by rotation*, or the *Barnett effect;* and (2) *rotation by magnetization*, or the *Einstein-de Haas effect.*

Much the most extensive measurements have been made in my own laboratories, in two series, as follows: one series (I) by means of my own effect (1920–1925),[2, 3] the other series (II) by means of the Einstein-de Haas effect (1925–1939).[3] The results of the two investigations agree, for the most part, very well; but those of the second are much the more precise, partly because the sources of error in working with the Einstein-de Haas effect, while themselves quite difficult to eliminate thoroughly, are not so difficult to eliminate as are those encountered in working with the other effect. The evidence appears to show definitely that the results of Series II are the most precise which have hitherto been obtained.

On account of such discrepancies as exist between the results of Series I and Series II it has long seemed important to make a new and independent investigation of the rotors used in Series I by means of the same effect, even if increased precision should be impracticable to attain. The first part of such an investigation is described in this paper. Fortunately, for most of the rotors, the precision has already been considerably increased over that of Series I.

At the same time it is important to study the various methods of investigation further with the hope of increasing the precision. For the pure magnetic metals and alloys to which my own work has been restricted, the values of the gyromagnetic ratio extend through only the small range from about $m/e$ x 1.00 to about $m/e$ x 1.08 or 1.09, and only two of them are known to less than one per cent. Furthermore, as Professor Pierre Weiss remarked to me at the Strasbourg *Réunion*,[3] the work "must be extended to more

[1] Ohio Academy of Sciences and American Physical Society, November and December, 1914. See S. J. Barnett, *Magnetization by rotation*, Phys. Review 6: 239–270, 1915.

[2] S. J. and L. J. H. Barnett, Proc. Amer. Acad. 60: 127–216, 1925.

[3] See especially S. J. Barnett, *Gyromagnetic ratios for ferromagnetic substances: New determinations and a new discussion of earlier determinations.* Proc. Amer. Acad. 73, 401–455, 1940. See also a briefer discussion in the reports of the *Reunion d'Études sur le Magnetisme*, held at Strasbourg under the presidency of P. Weiss in 1939. These reports were published in Paris by the *Institut International de Cooperation Intellectuelle* and the *Service Central de la Récherche*

substances and to the same substances in different states; and for this purpose it is necessary to increase the precision."

Series I was made by a magnetometer method in a building free from iron, a part of the plant of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, where the magnetometer could be controlled without too great difficulty. Experience in the Norman Bridge Laboratory, which is full of iron, involving the use of a magnetometer in another investigation,[4] convinced me that in this laboratory the method of electromagnetic induction would be preferable, if it could be made sufficiently sensitive, on account of the superior degree of freedom from asymmetry—the source of most of the errors—which it makes possible. At the same time it offers the great advantage over the magnetometer method that in it provision can far more readily be made to measure only changes of magnetization instead of magnetization itself. Furthermore, for the sake of a greater degree of independence, it was desirable to adopt a method radically different from that used in Washington.

The new work is still far from complete, progress having been greatly interfered with by the defense and war situations. While it is certain that the work can be greatly improved in many particulars, nevertheless a degree of precision at first hardly hoped for by the method of investigation here adopted has already been achieved in this preliminary work, and important independent confirmations of earlier work obtained. For this reason, and because the investigation cannot probably be greatly advanced until after the war is over, publication of the work already accomplished is being made at this time.

2. Principal sources of error associated with the rotation in the method of electromagnetic induction. Most of the sources of error so far discovered in the study of magnetization by rotation, together with methods designed to eliminate them, are discussed in the earlier papers. Some of these will be treated further, and others will be discussed, in this paper. The sources of error directly associated with the rotation may be divided into two classes as follows:

A. *Sources independent of the rotor's magnetization*, viz.: (1) eddy currents arising from incomplete compensation of the earth's magnetic field, (2) electric currents produced by thermal effects at the bearings and by air driven by the moving rotor against the adjacent parts of the apparatus, (3) electrical or magnetic effects in the motor and other driving apparatus and auxiliary apparatus.

B. *Sources dependent on the rotor's magnetization*, viz.: (1) torsion of the rotor, driven from one end and moving with friction in a bearing at the other, (2) thermal effects on the rotor due to friction at the bearings and to air currents, (3) centrifugal expansion, and other strains, produced by the rotation and perhaps by gravitation, (4) axial displacement of the rotor produced by the rotation, (5) displacement of the rotor in altitude or azimuth, or distortion, in the incompletely compensated magnetic field of the earth, (6) non-uniform initial position-angle setting of the rotor, (7) vibration.

3. The method of electromagnetic induction as applied to the study of magnetization by rotation. In the method of electromagnetic induction a coaxial coil of insulated wire surrounds the rod under investigation and is connected in circuit with a galvanometer of the fluxmeter type. If the magnetic moment of the rotor is changed by any small amount the fluxmeter gives a deflection proportional thereto. In the main experiment the angular velocity of the rotor is changed by the amount $\delta\Omega$ and a change of magnetic moment $\delta M$ results, producing a deflection $D$ of the fluxmeter. In a calibrating experiment a minute axial magnetic intensity is applied to the rotor and is altered by an amount $\delta H$. This produces a change $\delta M'$ in the moment, which causes a deflection $\Delta$ of the fluxmeter. The quantities $\delta M$ and $\delta M'$ are proportional, respectively, to the change $\rho\delta\Omega$ (where $\rho$ is the gyromagnetic ratio of the magnetic element) in the intrinsic magnetic intensity of rotation and the change $\delta H$ in the intensity of the axial field. Thus we have

$$\frac{\delta M}{\delta M'} = \frac{\rho\delta\Omega}{\delta H} = \frac{D}{\Delta}$$

whence

$$\rho = \frac{D}{\Delta} \cdot \frac{\delta H}{\rho\Omega} \quad . \quad . \quad . \quad . \quad (3\text{—}1)$$

In the actual earlier experiments by this method *two similar* rods were mounted with their axes horizontal and normal to the magnetic meridian, and *two similar* coils of insulated copper

*Scientifique de France*, under whose auspices the *Réunion* was held, but their general distribution has hitherto been prevented by the war.

[4] Proc. Amer. Acad. 68: 229, 1933.

wire were mounted symmetrically about their centers, as shown in Fig. 3—1. These coils were placed in series with one another and with the fluxmeter and were oppositely connected so that any fluctuations in the intensity of the earth's magnetic field, which act in the same way on both rods, might produce no effect on the fluxmeter. One of the rods, the *compensator*, *A*, remained at rest; while the other, the *rotor*, *B*, was alternately rotated in opposite directions, the change in its magnetization being determined as it came to rest or to very low speed. For use in the standardizing experiments the rods *A* and *B* were uniformly wound with single layer coils of insulated copper wire; and two long wooden rods

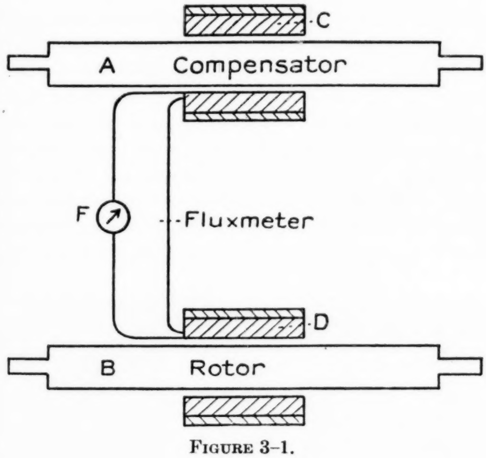


FIGURE 3-1.

of the same diameter and similarly wound were provided for attachment to the ends of the rotor, in order to make reductions to a strictly uniform field. The earth's field, in the region occupied by the rotor, was, in general, neutralized in order to prevent eddy currents and changes in its axial flux due to alterations in the shape or position of the rod, including possible slight alterations of axial orientation caused by the rotation. In alternate runs the rotor was rotated in opposite directions at the same speed in order to eliminate the effects of changes of magnetization due to centrifugal expansion and other disturbing effects. To eliminate the effects of torsion, observations were made for the two orientations of the rotor's axis, 180° apart. The mean result is free from torsional error if the frictional torques at the two ends of the rotor are exactly identical. Unless the bearings and journals at the two ends are exactly similar, complete elimination of the error would necessitate that the bearings also be reversed—a precaution not taken in any of the work; and if the centrifugal thrusts on the bearings due to axial asymmetry of the rotor are different at the two ends, complete elimination of the error would necessitate also that the rotor be driven alternately from the two ends, instead of only from one end, with a single motor. This precaution has been taken only in a part of the new work. Such interchange of motors may help, furthermore, to eliminate the possible error due to differential longitudinal shift of a rotor on reversal of its direction of rotation.

4. MECHANICAL ARRANGEMENTS IN THE NEW WORK WITH ROTOR AXIS HORIZONTAL AND ROTOR AXIS VERTICAL. In the greater part of the new work the rotor, compensator, etc. were mounted with axes horizontal, as in all the work done hitherto. In the remainder of the work the axis of rotation was vertical.

In all the earlier work the rotor was always directly driven by a single countershaft, or a series of countershafts, permanently located either east or west of it. To eliminate certain errors, especially that due to torsion, it was therefore necessary to reverse the rotor's azimuth periodically—a not very troublesome operation, but one requiring time and introducing some uncertainty because of the possible changes in the rotor's magnetization incurred in the process.

Primarily in order to effect the same elimination, in that part of the present work in which the rotor was mounted with the axis horizontal, it was located symmetrically between two exactly similar motors at equal distances, and was driven at will by either one through a series of four countershafts. The motors are of the synchronous type and were always driven at the same speed, viz. 60 r. p. s. In general equal numbers of observations were made with the two motors.

The countershafts were simple bakelite tubes of various sorts, with internal diameter one-half inch and walls one-sixteenth, three thirty-seconds and one-eighth inch thick. They ranged in length from about one foot to about two feet. Each of the tubes nearest the rotor was provided with a brass terminal piece centrally drilled to fit the rotor journals. It was attached to the tube by two similar screws at opposite ends of a diameter, and to the rotor by a set-screw and nut, with a counterbalance on the other side. The

terminal pieces used in much of the work ended in circular flanges which were drilled with sixteen holes uniformly spaced around the edge, to serve as position angle indicators. The other end of

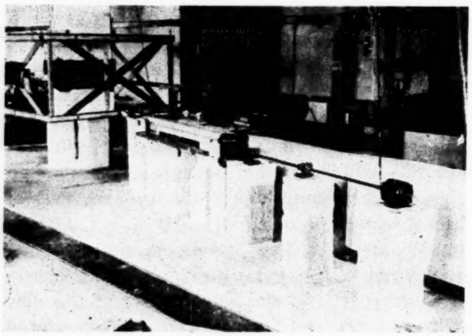

FIGURE 4-1. When this photograph was taken the west motor etc. had been removed to make room for the vertical drips arrangements, which are shown in place on the left.

the tube, in the earlier part of the work, slipped over a half-inch journal, to which it was fastened with small screws when in operation.

This journal was part of a bronze thrust bearing permitting only very slight longitudinal motion. When the screws were removed, the tube could be slipped farther on to the journal, freeing the rotor entirely from the tube. In later work, in order to reduce longitudinal play still further, each of these cylindrical thrust bearings was replaced by a combination ball and cylinder bearing. Only one of these ball and cylinder bearings was used with vertical drive. The cylinder was of bronze. The balls and rings were of steel, but were too small (and too well demagnetized) to have any appreciable effect on the work. The other journals of the countershaft system were of bronze or stainless steel, one-half inch in diameter, and ran in bearings of babbitt metal in bronze blocks. All the bearing blocks were mounted either on the bed plate carrying the rotor, on on separate piers of concrete made from white limestone and white Atlas cement.

For horizontal drive, the motors were mounted on separate concrete piers and drove the countershafts directly at 60 r. p. s. The rotor was halfway between the motors, which were nearly nineteen feet apart.

The bakelite tubes are insulators so that no eddy currents were formed in them; also no currents of thermal origin could traverse them. Their lack of great torsional stiffness helped to prevent too great initial acceleration of the rotor, and their flexibility made it unnecessary to take the greatest pains to secure exact alignment. They are also so light that mechanical disturbances when they do not run true are at a minimum.

While the Washington work (I) was in progress, Mr. C. A. Kotterman, now of the Bell Telephone Laboratories, who acted as research assistant, suggested that in order to eliminate friction at the end of the rotor more remote from the motor we use vertical drive instead of horizontal drive. The effect of the torsion produced by this friction was small, however, and was eliminated without great difficulty. At the same time the change to vertical drive would have required a great deal of labor and expense. Hence the change was not made. In the course of the new experiments with horizontal drive, however, it appeared that some of the disturbances might be due to flexure of the rotor by the earth's gravitational field; and in order to avoid this and at the same time reduce torsion it was concluded to adopt Mr. Kotterman's suggestion and modify the apparatus to make vertical drive possible. In the arrangement adopted the rotor was driven from above

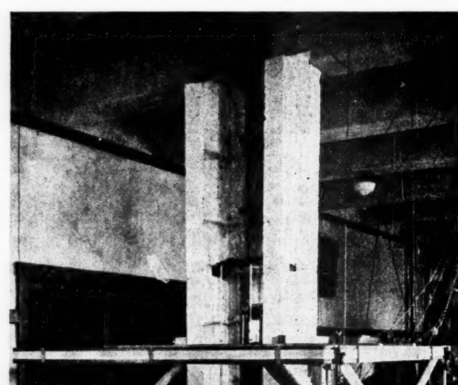

FIGURE 4-2.

only by a single motor, so that the possible torsional effect due to centrifugal thrust of the lower journal against its bearing was not eliminated.

In all the new work the rotors were mounted on the small bed-plate used before, and in bearings which are similar except that bearing bronze is substituted for the agate mostly used before. In a good deal of the work the small cylinders acting as bearing pieces have been slotted, and the clearance between journal and bearing piece adjusted when desirable by means of screws set into the heads of the bed-plate and acting radially at both sides of the slot.

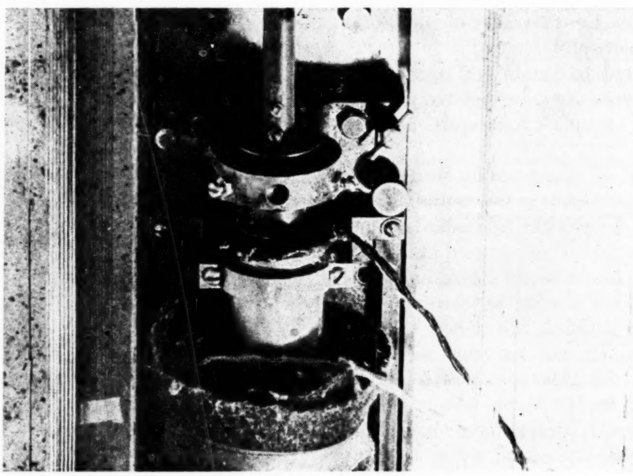

FIGURE 4–3.

In the work with axis horizontal the small rotor bed-plate was mounted on the same heavy bronze bed-plate used for this purpose in Washington; while in the work with axis vertical the rotor bed-plate, the motor and all the extra bearings were mounted on a new long and heavy combination brass-and-bronze bed-plate bolted to a heavy vertical concrete pier resting on the floor, and to a heavy steel beam in the ceiling of the laboratory. See Figs. 4—1 and 4—2.

In Fig. 4—3 the test coil, heavily insulated with felt, is shown mounted in position for vertical drive.

5. THE ROTORS. Most of the rotors are the rotors used in Washington. Three new rotors have been constructed, and one rotor used earlier reconstructed. One of the new rotors is Electrolytic Iron IV. This is from a part of the same rod of iron from which Electrolytic Iron II was formed. Material was lacking to make the magnetic part of this rotor as long as that of the others, and the defect was made up by extending the length of the bronze journal pieces. The diameter is 3.0 cm. and the length of the magnetic material 20.5 cm.

Another new rotor is Electrolytic Iron III. It was cut from a new rod of Dr. Yensen's electrolytic iron, and is 3.0 cm. in diameter.

A third new rotor is Nickel IV, cut from a rod obtained from the International Nickel Co.

Finally, the old rotor Cobalt I, used in Columbus and Washington, was provided with terminals similar to those on the other rotors. The later and better type of construction used in Washington was adopted.

6. MOTORS, SPEEDS, AND CORRECTION FOR IMPERFECT FLUXMETER ACTION. In the earlier work the accurate determination of speeds and the requirement of strictly identical speeds for the two directions of rotation, or of correction for the difference, made a great deal of labor necessary. All this vanished in the new work because alternating current power with the constant and precise frequency 60 cycles per second was always available, and synchronous motors could be used to obtain exactly 60 revolutions per second. Additional speeds were not used.

The motors are 1/8 h. p., self-starting synchronous motors by the General Electric Company. When the unmodified motor is used to drive the equipment, full speed is attained in about 0.5 or 0.6 second after the circuit is closed.

It is highly desirable that this starting interval be as short as possible in order that the galvanometer readings may be interpreted correctly. Nevertheless, when the observations began, it was feared that the acceleration corresponding to the time mentioned would be too great, perhaps for both magnetic and mechanical reasons. Hence an inertia wheel was constructed, in three parts. The first, $A$, is a heavy rolled brass disk provided with an axle and set-screw for attachment to the motor like a pulley. The second, $B$, is a ring of the same material which can be screwed onto one side of the first; and the third, $C$, a similar ring which can be screwed onto the other side of the first. The rings fit accurately over short hubs turned on the two sides of $A$, so that balancing is automatically secured.

The times required to attain full speed with one, two, and three discs, respectively, were approximately 1.1 second, 1.8 seconds, and 2.8 seconds.

While a number of observations were made with one and two discs loading the motors, it was found that all the discs could be discarded; and in most of the work they were not used at all.

It was at first supposed that a correction to the galvanometer readings might be necessary because the fluxmeter action is not perfect and the different effects which act on the secondary circuit, and which the galvanometers integrate, are not completed in the same time. Special time-lag experiments, however, have shown that no correction is necessary except when as many as two discs load the motor. It was estimated that in this case a (negative) correction of about one-fourth per cent should be made to the apparent values of $\rho$; and this correction has been applied to the relatively few results obtained with two discs.

7. Neutralization of the earth's magnetic field. If the rotor were strictly cylindrical and homogeneous, and if the surrounding secondary coil were truly cylindrical and regularly wound with its wires truly circular, and were mounted coaxial and otherwise symmetrical about the rotor, then eddy currents due to the rotation of the rotor in the earth's field would produce no resultant flux through the secondary coil, and it would thus be unnecessary (on this account) to annul the field. Partly because these conditions were not completely realized, the chief experiments were made with the earth's mean intensity in the region occupied by the rotor annulled as far as was practicable.

An additional reason for annulling the earth's field is to prevent any change of flux through the rotor due either to a minute change in its altitude or azimuth made possible by the existence of a small amount of bearing clearance, to flexure produced by gravitation or centrifugal action, or to lack of magnetic symmetry.

In the earlier part of the work with axis horizontal, the earth's magnetic field was annulled by two sets of coils of insulated copper wire. The principal set consisted of two pairs of long rectangular coils mounted with their wires (except for the ends of one coil pair) horizontal and symmetrical about the rotor, and thus normal to the magnetic meridian. The coils

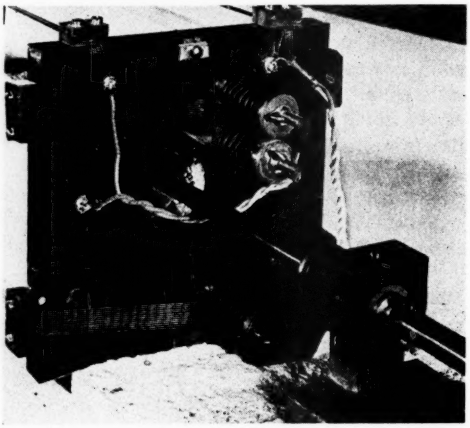

Figure 7–1.

were all somewhat more than 2.1 m. long, and all were about 25.5 cm. wide (See Figs. 4—1 and 7—1). The mean planes of the coils which annulled the vertical component of the earth's intensity were about 16 cm. apart, and those of the coil which annulled the horizontal component about 17 cm. apart. They were wound in grooves 1/16 inch apart (center to center), cut accurately in square bakelite blocks 25.5 cm. on each side and 2.5 cm. thick. Each coil was kept stretched tight by means of two springs whose tension could be adjusted as shown in Fig. 4—1. The coil pair which annulled the vertical intensity had 26 turns, the other 12 turns. The other set of coils consisted of the large square Helmholtz pair used in the work on the Einstein-de Haas effect, which hung nearby. This coil-pair was connected in parallel with the other vertical-intensity rectangular coil and, as it happened,

made the residual field at the rotor more nearly uniform than it would have been without it.

In the later part of the work with axis horizontal, the large Helmholtz pair was disconnected and the copper wires of the other coils were replaced by No. 20 bare spring brass wires. In each coil the spacing of the grooves prevented the individual wires from touching their neighbors. Vibration of the wires was prevented by the application of small pieces of surgeon's tape.

When the apparatus was mounted with axis vertical, three coils were used to neutralize the earth's magnetic field. See Fig. 4—2. The vertical intensity coil was the coil used for the same purpose in the Einstein-de Haas investigation, which was moved from its old position and mounted symmetrically about the center of the rotor. It was held in position by vertical brass wires attached to the ceiling and horizontal wooden rods connected with the pier. The N-S and E-W components of the horizontal intensity were neutralized by two long coils constructed much like those used in the latter part of the work with axis horizontal. The E-W coil had only two turns in each half; the N-S coil had eight. The bakelite end pieces were securely fastened to the concrete above and below by pieces of brass and bronze; and the wires were held taut by set screws, working in bakelite, which pressed against the upper ends of the loops. By means of the compensating coils the earth's mean intensity in the region occupied by the rotor was reduced to a small fraction of one per cent.

8. Test, compensating, and calibrating coils. In the work with axis horizontal two pairs of test coils were used, of nearly the same dimensions and with nearly the same numbers of turns. They were mounted as in Fig. 3—1. One pair was wound on bakelite bobbins, the other on brass bobbins. The coils of the first pair were machine wound by the Inca Co., but the windings were not very regular. The coils of the second pair were wound with great regularity by the Hollywood Transformer Co., and were used in the greater part of the work. These coils were six inches long, ½ inch thick, and had internal diameters of 1⅞ inches. Each contained a little more than 14,000 turns of No. 28 enameled copper wire. One coil served as rotor or test coil, and the other as compensator coil.

Three almost exactly similar calibrating coils were wound, one on a bakelite tube, the others on brass tubes. In the case of the last two coils great care was taken to see that the insulation was always perfect (except at one end, where the wire was usually attached to the tube). The first coil was used chiefly in the earlier part of the work. For details of the windings see § 17.

The test coil was mounted with precision, in brass clamps, coaxial with the rotor on the small bed-plate, and the calibrating coil was mounted coaxial inside the bobbin of the test coil. The centers of both coils and the rotor were practically coincident.

In the work with axis vertical, after a few observations made with one of the Hollywood coils as compensator, mounted on the laboratory wall, it was replaced by a large Helmholtz pair centered on the rotor. See Figs. 4—2 and 8—1.

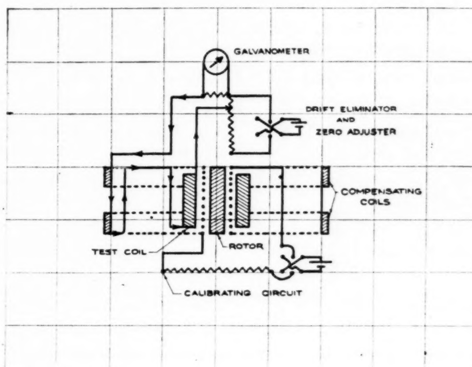


Figure 8–1.

Each coil of the pair was nearly square, about 2 ft. on the side, 4 in. wide, and 1 in. thick. The complete coil was wound with 2000 turns of No. 20 insulated copper wire in sections—each divided equally between the two halves, except for a few turns, permitting any even number of turns to be used from 2 to 2000. A special switchboard was provided for making the connections. The coil and board were electrically screened and thermally insulated with felt.

In addition to the great advantage of having the compensating coil centered on the rotor and its coil, there are the additional advantages of low resistance and of being able to determine with ease the proper number of turns for exact compensation for each rotor.

An approximate determination can be made by adjusting the current in the vertical earth intensity coil until the vertical intensity is annulled, then adjusting the number of turns until,

with the secondary circuit closed through a galvanometer, no deflection occurs when a slight change is made in the current of the vertical intensity coil.

If the field of the vertical intensity coil were uniform throughout the extent of the compensating coil, this determination would be exact. The correction which must be applied to make it exact was readily found by subsidiary experiments made with the help of additional coils of the proper shapes but smaller dimensions. It was found that the number of turns determined as above must be multiplied by 1.007.

It was found necessary to screen electrically the whole secondary circuit. Much of it was insulated thermally.

9. THE GALVANOMETERS, PHOTOELECTRIC CELLS, ETC. In order to attain adequate sensitivity, two galvanometers were used in conjunction with either two similar photoelectric cells or a Moll thermo-relay, which was used in a relatively small part of the work.* The first galvanometer had the characteristics of a fluxmeter. The optical arrangements finally adopted were as follows (See Fig. 9—1):

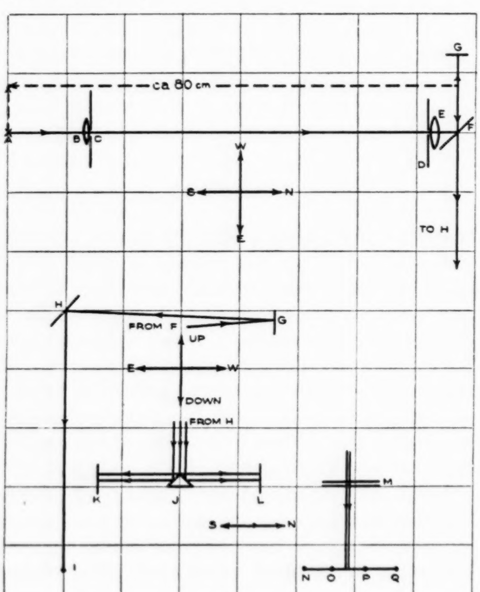


FIGURE 9–1.

* I am indebted to Dr. Yensen and Dr. Goetz for information with regard to the photoelectric cells, and to the latter for the loan of the Moll relay.

A beam of light from the filament $A$ of an electric lamp passed in a horizontal plane from $S$. to $N$ through a short-focus convex lens $B$ and a rectangular opening $C$ in a thin brass plate in contact therewith, and onward through a circular opening $D$ and a long-focus convex lens $E$ to a plane mirror $F$. This mirror sent the beam westward and slightly upward until it fell on the plane mirror $G$ of the galvanometer in circuit with the rotor and compensator coils. An image of a part of the filament was formed on the mirror $G$. From this mirror the beam was reflected eastward and slightly upward. It fell on a plane mirror $H$, which reflected it vertically downward. An image of the rectangular opening $C$ was formed in a fixed position $I$ if there was no current through the galvanometer and if no further provisions were made.

When the photoelectric cells were used (Fig. 9—1) the beam was divided into two equal parts by reflection from a right angled prism $J$. An image of half of the opening $C$ was formed on the face of each of the cells, $K$ and $L$. When the galvanometer mirror moved, the upper edge of each half image remained essentially fixed while the two lower edges moved in opposite directions equally. When the Moll relay was used the vertical beam passed through a cylindrical lens $M$, which concentrated the light from $C$, made narrow for this purpose, on or near the center of the thermocouple $NOPQ$.

With a galvanometer having the characteristics of a fluxmeter there is of course, in general, a more or less slow drift, which it is often necessary to compensate. This was accomplished, when necessary, as in the work of 1914 and 1915, by introducing a minute extraneous electromotive force into the circuit. A single dry cell was connected through a reversing switch to the terminals of the high resistance of an Ayrton shunt. Leads were connected from the variable resistance terminals through an adjustable resistance box to two points, with a very low resistance between them, in the galvanometer circuit. This arrangement not only serves to eliminate the drift but provides a useful device to control the galvanometer while making adjustments.

The mirrors $F$ and $H$ and the lenses $E$ and $M$ were provided with all neeessary adjustments; and the prism $J$, or the Moll relay $NQ$, was mounted on a small table which could be moved northward or southward by means of a fine micrometer screw.

When the photoelectric cells were used, they

were connected in opposition to one another, and either in series or in parallel, in closed circuit with the second galvanometer, suitably shunted to produce critical or approximately critical damping. When the Moll relay was used its terminals $NQ$ were connected to the galvanometer terminals through a series resistance sufficient to produce critical damping.

The galvanometers with their support and all the optical devices, except $A$, $B$, and $C$ were mounted inside a box of wood internally blackened and sufficiently tight to prevent trouble from convection currents. Internal blackened screens were also provided to prevent extraneous light from reaching the photoelectric cells and to limit the beam where necessary. The lens $B$ and the brass plate with opening $C$ were mounted at the end of a large bakelite tube supported on a concrete block and projecting through a wall of the box without contact. The opening between the two was screened with black cloth. The lamp $A$ was mounted on a separate concrete pier.

10. The galvanometer support. Since the angular motion of the beam of light after reflection from the mirror of galvanometer $G$ is very minute, it was imperative that the galvanometer be mounted on a support as free from vibration as possible. Four thin brass rods were mounted vertical, or nearly vertical, in four brass castings resting firmly on a heavy concrete shelf. A horizontal rectangular wooden board was fastened to the tops of these rods near its corners. From points of this board, also near the corners, but nearer to the center than the rods, four vertical brass tubes proceeded downward. At the bottom, a little above the stone shelf, they carried a second board similar to the first; and at a level about three quarters of the distance from the top to the bottom they carried a third and heavier board. Across the central part of the top of this was screwed a still heavier board on which both galvanometers were mounted. A number of pieces of angle iron were screwed to the boards to prevent their warping. The lower board carried heavy weights, and on top of the upper board were pans of oil to provide internal damping. The masses were all arranged as symmetrically as practicable about a vertical line through the center. The arrangement formed a heavily damped inverted pendulum, and behaved much like the apparatus devised for the same purpose by R. Müller. In Müller's apparatus, however, there were only three rods, instead of four, so that horizontal motions are much more likely to produce angular motion of the suspended system.

11. Elimination of eddy current effects. Tests for eddy current effects were made in several different ways, as follows:

I. As in the Washington investigation, numerous measurements were made with a copper rotor[5] of nearly the same dimensions as the larger magnetic rotors, similarly placed and rotating both in the earth's unaltered field and in the compensated fields with the same speed of 60 r. p. s. The first experiments were made with the Inca coils in use. In the uncompensated field reversal of rotation gave a mean deflection of + 4.44 cm. when the coil was so set that a mark on its bobbin was as far south as possible. When the coil was set with the mark north, the corresponding deflection was − 1.09 cm. When the field was compensated the corresponding deflections were + 0.86 cm. and + 0.10 cm. The sensitivity was such that the same reversal of rotation of the permalloy rotor would have given the deflection − 15 cm. Thus with the mean field annulled and the coil set at the position angle giving the less effect, the apparent eddy current effect in copper was about 1 part in 150 of the gyromagnetic effect in permalloy and in the opposite direction, indicating that the *apparent* gyromagnetic ratio should be slightly increased. As shown in earlier papers, however, the eddy current effect in the magnetic rotors should be much less than that in copper, and thus much less than 1 part in 150 in the case of the permalloy rotor, with the coil set as above.

In the case of the Hollywood Transformer Company's coils with axis horizontal, the mean eddy current deflection for copper in the compensated field was less than + 0.1 mm. when the sensitivity was such that the gyromagnetic effect in Cobalt II was about + 6.3 cm. The deflection was practically independent of the position angle of the coil (over a range of 270°). When the apparatus was mounted with axis vertical, the copper deflection was − 0.003 cm. when the gyromagnetic deflection for the rotor Fe IV was about − 4.5 cm.

II. Direct experiments on the eddy current effects in the full uncompensated field of the earth were made on two of the magnetic rotors,

[5] The copper rotor had journals which were not insulated from the main body of the rotor. Hence it is possible that a part of the copper effect may originate from thermal *e. m. f. s* at the (bronze) bearings.

viz., Permalloy and Norway Iron. The Permalloy was rotated both in the neutral field, in the original uncompensated field, and in the original field approximately reversed. The Norway iron was rotated in the original field and in the original field reversed. The experiments showed that the *full original* field *reduced* the apparent gyromagnetic ratios by the fraction $0.025 \pm 0.005$.

III. In a few of the gyromagnetic experiments made with the Inca coils, the rotor coil was mounted with the mark in one of its two positions only. In most of the work with these coils, however, observations in nearly equal numbers were taken with the coil set at this position angle and the angle differing from this by 180°. In one case the eddy current effect, if any, should be many times greater than in the other. For the seven rotors investigated in this group (Group I) the difference between the apparent values of $\rho$ for the two cases was only the fraction $0.003 \pm 0.012$ of $\rho$, the average error being thus four times as great as the apparent difference between the two eddy current effects. No correction, was made to the results obtained in either way.

IV. In most of the horizontal drive experiments made with the Hollywood coils, still another test was applied. Half the rotations were made with a mark on one end of the bobbin east, and half with the coil reversed in azimuth and the mark west. The mean difference found for $\rho_E - \rho_W$ was $-0.002 \pm 0.015$.

12. THE POSITION ANGLE ERROR AND ITS ELIMINATION. If the coil, the rotor, and its magnetization possessed perfect axial symmetry, and the various effects above considered vanished, no change of flux through the secondary circuit would occur when the rotor is turned from one position angle to another. On account of asymmetry, however, such changes usually occur. If the rotor is set at a given position angle and then driven up to full speed (or any but a very small speed) the galvanometer gives a deflection $P$ corresponding to the change of flux from its initial value to its mean value; and this effect is added to all the other effects involved. Since it is independent of the direction of rotation it is eliminated from the final result if the rotor is set initially at the same position angle for both directions of rotation. The deflection $P$ retains very nearly the same magnitude, but changes its sign, when the initial position angle is altered by 180°. To make the elimination more certainly complete, each group of observations mark $U$ (or $W$) and mark $D$ (or $E$) in the standard sets was made in two parts, (1) and (2), the initial position angles in the two differing by 180°. Also, the initial position angles were in general, but not always, so chosen that the deflection $P$ was a small part of the gyromagnetic deflection $R$. In some cases $P/R$ was greater than unity.

The effect under discussion was usually investigated for the various rotors by observations independent of the main rotation experiments. In the method frequently employed the rotor was repeatedly set initially at a given position angle, and then turned suddenly in separate experiments from this initial angle to each of a series of angles differing from this by intervals up to nearly 360°. A rough curve was platted between the deflection and the increment of the position angle.

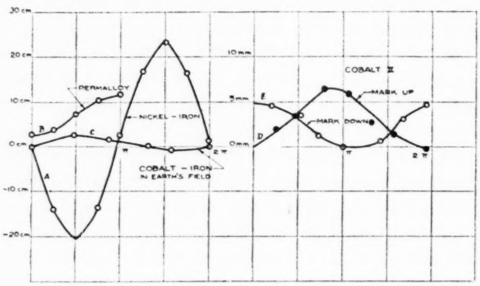


FIGURE 12–1. FIGURE 12–2.

In the main experiments the rotor was then set at the position angles corresponding to the mean deflection. This makes $P$ zero or a minimum.

Curves of this sort are given for several rotors in Fig. 12—1 and Fig. 12—2. All the curves are for axis horizontal except that for Cobalt-Iron, which was mounted with axis vertical. The curve shown for this rotor was obtained when only the vertical component of the earth's magnetic intensity was annulled. This rotor, and some of the others, gave effects too small to measure readily when mounted with axes vertical in a completely neutral field.

13. EFFECTS OF MOTORS AND BEARINGS. Inasmuch as the motors and ball bearings are made of iron they doubtless alter slightly the magnetic intensity of the field acting on the rotor and its coil. There was no reason, however, to expect the rotation of motor and bearing parts to produce any mean effect; and especially there was no reason to expect the *reversal* of their rotations to produce any effect.

Furthermore, direct experiments made with the rotor and compensator at rest and in their

standard positions, but with the motor and all the other driving mechanism in rotation except the countershaft nearest the rotor, have shown that these rotating parts produce no certain effect. No effect of the countershafts nearest the rotor was to be feared because they are made of bakelite tubes with only small brass plugs at the ends nearest the rotor. Thus with the Cobalt-Iron rotor in position and at rest, reversal of the motor etc. gave the mean deflection $-0.015 \pm 0.011$ cm. with sensitivity such that a deflection of $-8$ cm. would have been produced by a corresponding reversal of the rotor's own rotation.

14. Effects of radial forces on rotors. Rough tests were made on several of the rotors set at fixed position angles, to see what would happen in both compensated and uncompensated fields when either end of the rotor was pushed strongly by hand (at a point just outside the bearing) northward and southward, or upward and downward. In one case the middle of the rotor also was pushed. For Cobalt II and Electrolytic Iron IV, which showed very large effects of longitudinal motion, the deflections were only small fractions of the gyromagnetic deflections; likewise for Cobalt I. For Nickel I the deflections were of the same order as the gyromagnetic deflection, but in most cases smaller. For Nickel IV the effects were large, some of them much larger than the gyromagnetic deflection. The effects vary in sign and magnitude with the orientation of the axis of the rotor, with the position angle, with the direction of push, and with the magnetic field, and undoubtedly with other factors, and in such a complicated way as to defy a satisfactory analysis at the present time.

If such effects are produced in the principal experiments on account of asymmetry, by the rotation of the rotor, as they doubtless are, they can have only a small effect on the determination of the gyromagnetic ratios, because they must be nearly identical for both directions of rotation. The only reason they may not be exactly identical is that the rotor journals must move laterally to slightly different positions for the two directions of rotation.

These effects doubtless enter into the quantity designated below as $M$, which is calculated on the assumption that it is proportional to an even power of the speed, and so independent of the direction of rotation.

If any part of the effect is proportional to the speed, it is probably partly eliminated as the torsion effect is eliminated by reversing the rotor in its bearings and by reversing the drive from one end to the other. The effects usually reversed in this manner and designated below by $T$ cannot be due entirely to torsion, because they are in at least some cases largely independent of the rotor's residual magnetic moment, in both sign and magnitude.

15. Errors due to axial shift of the rotors. When a rotor is driven either right-handed or left-handed, it is possible for several reasons that a longitudinal shift $R$ or $L$ may occur; and the shift may be different for the two directions of rotation. If $R$-$L$ is not zero a systematic error may thus be introduced.

On this account three series of measurements of longitudinal displacements were made. The first series was made with a telescope and scale while the cylindrical bearings were in use (Group I). With the right motor in operation, no differential effect was observed; with the left motor, a displacement of possibly 1/20 mm. at first occurred. This disappeared, however, when the bearings were oiled, as they were oiled in the gyromagnetic experiments proper. In this case $R$ and $L$ were both zero, and it was estimated that a shift of 1/100 mm. would have been detected.

The second series was made with a microscope after the ball bearings had been installed for horizontal drive (Group II). Probably 0.003 mm. would have been observed, but no shift was detectable. Only one of the two similar sets of bearings was investigated.

The third series was made after the apparatus had been mounted for vertical drive (Group III), with a long range microscope especially constructed for the purpose. Several rotors were used. For Electrolytic Iron III, Permalloy, and Steel IV (a small rotor) the mean measured shifts ($R$-$L$) were $+0.002$ mm., $+0.002$ mm. (and again $-0.001$ mm.), and $-0.002$ mm., respectively.

Supplementing these measurements, many others were made on the galvanometer deflections produced by moving the rotors longitudinally in their bearings through measured distances of a few millimeters. In each case the mean deflection $D$ produced by moving the rotor eastward (or downward in the case of vertical drive) through a distance of 1/100 mm. was calculated, together with the ratio of $D$ to the gyromagnetic deflection $R$ produced by driving the rotor up to full speed in the right-handed or

TABLE 15—1

EFFECTS OF AXIAL SHIFTS OF ROTORS

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Rotor | Group | $I \times 10^3$ | $\Delta$ | $\Delta'$ |
| | | e. m. u. | | |
| Fe III | I | +0.6 | +0.004 | +0.004 |
| | III | −0.2 | +0.005 | −0.002 |
| | | +0.2 | +0.006 | +0.002 |
| Fe IV | II | −0.3 | −0.024 | −0.024 |
| | III | −0.3 | −0.020 | −0.023 |
| | | +0.3 | −0.018 | −0.025 |
| | | +1.2* | −0.003 | −0.003 |
| | | −1.2 | −0.009 | −0.009 |
| | | +1.2 | −0.009 | −0.006 |
| | | −0.2* | −0.002 | |
| | | +0.2 | −0.002 | |
| | | +0.2 | −0.003‡ | |
| | | +0.2 | −0.003‡ | |
| Steel III | III | +0.5 | −0.004 | −0.005 |
| | | −0.5 | −0.005 | −0.008 |
| Ni I | II | −1.4 | +0.005 | +0.002† |
| | II | −0.3 | +0.005† | |
| | | +0.3 | +0.007† | |
| | III | −0.2 | +0.005 | |
| | | +0.2 | 0.000 | |

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Rotor | Group | $I \times 10^3$ | $\Delta$ | $\Delta'$ |
| | | e. m. u. | | |
| Co II | II | +1.6 | −0.012 | |
| | | +0.3 | −0.008 | |
| | | −0.3 | −0.012 | |
| | | +0.3 | −0.009 | |
| | | +0.6 | −0.010 | −0.011 |
| | | −0.4 | −0.016 | −0.017 |
| Hopk Al | III | −36 | +0.002 | +0.003‡ |
| | | +36 | −0.002 | |
| Permalloy | III | −1.8 | +0.003 | −0.002 |
| | | +1.8 | +0.004 | 0.000 |
| Cobalt Fe | III | +0.3 | +0.001 | 0.000 |
| | | −0.3 | +0.001 | −0.003 |
| Heusler Alloy | III | +0.2 | +0.005† | |
| | | −0.2 | +0.005† | |
| Ni IV | II | +2.1 | +0.038† | |
| | III | +0.13±* | +0.002 | |
| | | −0.13± | +0.001 | |
| | | −0.13± | 0.000 | +0.005 |
| Co I | II | −1.8 | −0.004 | −0.004 |
| | | +1.8 | −0.004 | −0.004 |

* Demagnetized with A. C.
† The record does not show whether the earth's field was compensated or not.
‡ Only the vertical component of the intensity ot the earth's field was annulled.

clockwise direction as seen from the west end in the case of horizontal drive (or from above in the case of vertical drive), at the same sensitivity. The mean values of this ratio, designated by $\Delta$ for the case in which the earth's field was compensated, and by $\Delta'$ for the case of uncompensated field, are given in Table 15—1.

These results, coupled with the observations on the actual longitudinal shifts of the rotors, appear to indicate that in at least most of these typical cases only negligible mean errors could have resulted from longitudinal shift. It was not practicable, however, to keep the bearings in exactly the same state at all times, so that appreciable errors doubtless sometimes occurred.

The magnitude of the quantity $\Delta$ is the fractional error in the magnitude of $\rho$ which would result from 1/100 mm. shift.

The quantities $I \times 10^3$ in column 3 are rough values of the mean intensity of magnetization of the rotors multiplied by 1000, the sign in each case indicating the polarity of the downward (or eastward) end.

The effects $\Delta$ and $\Delta'$ would be expected to vanish if the magnetization were symmetrical about the equatorial plane of the rotor and if the longitudinal gradient of the residual magnetic intensity were zero. The parts of $\Delta$ or $\Delta'$ due to the gradient should have the same sign for all shifts in Groups I and II, and likewise for all shifts in Group III, as in fact $\Delta$ and $\Delta'$ often do. The parts due to asymmetry might or might not change sign with the measured magnitude or sign of the magnetization. Both kinds of effect are apparent in the table. A complete analysis seems impossible.

16. Processes involved in the main experiments and the standardizing experiments. In the main experiment, idealized for the case in which no sources of error are present, two processes are involved, equivalent to the following:

(1) In the first process the angular velocity of a rotor is quickly raised from $O$ to $\Omega$ radians per second, and the galvanometer deflection $R$ due to the change of flux $\delta\varphi_R$ through the secondary circuit is read. The change in the moment of the given rotor in this process is

$$\delta M_R = C\rho\Omega \qquad (16\text{—}1)$$

where $C$ is a constant for the rotor. Then

$$\delta\varphi_R = {}_1K_R\delta M_R + {}_2K_R\delta M_R = K_R\delta M_R = K_R C\rho\Omega \qquad (16\text{—}2)$$

where ${}_1K_R$ and ${}_2K_R$, whose sum is $K_R$, are constants for the given rotor and rotor coil, and the rotor and compensating coil, respectively. Always ${}_2K_R / {}_1K_R << 1$, and ${}_2K_R$ is nearly identical for all rotors. For the deflection $R$ produced by $\delta\varphi_R$ we have

$$R = A\delta\varphi_R = AK_R C\rho\Omega \qquad (16\text{—}3)$$

where $A$ is constant for the secondary circuit used with the given rotor.

(2) In the second process a small current $i_s$ is sent through the short primary coil surrounding the rotor, and the deflection $S$ due to the change of flux $d\varphi_s$ is observed. In this case we have

$$S = A\delta\varphi_s = A\,({}_1K_s + {}_2K_s)\,\delta M_s = AK_s\delta M_s \qquad (16\text{—}4)$$

where $\delta M_s$ is the change in the moment of the rotor and coil together, and ${}_1K_s$ and ${}_2K_s$, whose sum is written $K_s$, are constants evidently nearly equal to ${}_1K_R$ and ${}_2K_R$, but not quite equal because the equivalent intensities of magnetization are not similarly distributed in (1) and (2).

In (1) the intrinsic intensity of rotation, viz. $H_\rho \equiv \rho\Omega$, is strictly uniform, while in (2) the field acting on the rotor is not uniform, nor is the field limited to the volume occupied by the rotor. It is necessary to find the deflection $\Delta$ (§ 3) which would be produced by a known *uniform intensity* $H_0$ acting *on the rotor alone.*

Consider therefore the following procedure. Imagine the solenoid, with diameter and number of turns per unit length unchanged, to be lengthened symmetrically about the rotor to such an extent that its field in the central section occupied by the rotor is essentially uniform, intensity $H_0 = 4\pi ni_s$. Suppose leads to be attached both at the (new) ends and at the points of the winding to which leads were attached before the length was increased.

If now the current $i_s$ were sent through the complete coil, with the rotor in position, we should obtain, instead of (16—4)

$$S' = A\delta\varphi_s' \qquad (16\text{—}5)$$

If the same current were sent through the coil with the rotor removed we should obtain

$$S'' = A\delta\varphi_s'' \qquad (16\text{—}6)$$

Hence,

$$S' - S''\ (\equiv \Delta, \S\, 3) = A\,(\delta\varphi_s' - \delta\varphi_s'') = AK_R CH_0 \qquad (16\text{—}7)$$

is the deflection produced when the intensity $H_0 = 4\pi ni_s$ acts on the rotor alone.

From (16—3) and (16—7) we find

$$\frac{\rho\Omega}{H_0} = \frac{R}{S' - S''} = \frac{R}{\Delta} \qquad (16\text{—}8)$$

It is, of course, not practicable to extend the rotor solenoid in situ as imagined above. For this and other reasons we proceed as follows:

We construct another solenoid, exactly like that imagined, and mount the rotor and the coils in the same positions with regard to it. We also replace the two galvanometer arrangement with a single galvanometer less sensitive than either of the other two, and observe the throws $D_0'$ and $D_0''$ (corresponding to $S'$ and $S''$) produced by reversing a current $I$, which is much larger than $i_s$ but still so small that the effects produced remain proportional to currents. In addition we observe the throw $D_0$ (corresponding to $S$) produced when the current $I$ is reversed in the central part of the long solenoid (identical in dimensions etc. with the short solenoid) while the rotor is in place.

We now have

$$D_0' - D_0'' = AK_R C \cdot 4\pi nI \qquad (16\text{—}9)$$

in place of (16—7); and

$$D_0 = A\delta\varphi' = AK_s\delta M' \qquad (16\text{—}10)$$

in place of (16—4).

From (16—7) and (16—9) we get

$$\frac{S' - S''}{D_0' - D_0''} = \frac{i_s}{I} \qquad (16\text{—}11)$$

and from (16—4) and (16—10) we get

$$\frac{S}{D_0} = \frac{i_s}{I} \qquad (16\text{—}12)$$

From the last two equations we get

$$S' - S'' \equiv \Delta = (D_0' - D_0'')\frac{S}{D_0} \qquad (16\text{—}13)$$

and thus

$$\frac{\rho\Omega}{H_0} \equiv \frac{R}{\Delta} = \frac{R}{S}\left[\frac{D_0}{D_0' - D_0''}\right] = \frac{R}{S}\,\alpha \qquad (16\text{—}14)$$

where $\alpha$ is written for the quantity within the brackets.

The values of $\alpha$ for the different rotors used in the investigation described here are given in Table 20—1 column 12. They are known to about 0.1 per cent, and are identical for horizontal and vertical drive, and for the Inca and Hollywood coils and the two compensating coils.

From (16—14) we obtain

$$\rho = \frac{H_0}{\Omega}\cdot\frac{R}{S}\cdot\alpha \qquad (16\text{—}15)$$

In the earlier work, both that by the method of electromagnetic induction and that by the magnetometer method, the calibrating deflection corresponding to $S$ was in general obtained for any set either before or after, or both before and after, the rotation experiments proper were completed, and with $\Omega$ either very small or zero. In some cases the calibration was made with the rotor at full speed, but little difference was found.

In this work, for a greater degree of certainty, the precaution has been taken, as indicated above, to measure $S$ and $R$ *simultaneously*. Switches are provided by which $S$ and $R$ can be given the same direction or opposite directions at will. In half the work $S + R \equiv A$, is obtained; in the other half, $S - R \equiv B$. Then $R/S = (A - B)/(A + B)$, or

$$R = S\,(A - B)/(A + B) \qquad (16\text{—}16)$$

From (16—15) and (16—16) we get

$$\rho = \frac{H_0}{\Omega}\cdot\frac{(A - B)}{(A + B)}\cdot\alpha \qquad (16\text{—}17)$$

for the final calculation of $\rho$.

An important reason for this procedure is that in the sensitive arrangement used in this work the sensitivity does not remain steadily constant, but often undergoes changes of a regular, or more or less irregular, character.

Another important reason for this procedure is that it eliminates effects of vibration.

17. THE CALCULATION OF THE GYROMAGNETIC RATIO. Both of the standardizing short solenoids wound on brass tubes (§ 8) had exactly the same diameters (about 41 mm. internal), lengths (about 28.5 cm.), and number $n$ of turns per cm. (44/2.540), the total number of turns in each being 494. They were lathe wound (with screw) from No. 28 cotton-enamel copper wire. The long solenoid was similar, but about 90 cm. long. Thus for each of these solenoids

$$4\pi n = \frac{4\pi \times 44}{2.540}$$

The short solenoid wound on bakelite, through an error on the part of the mechanician who constructed it, had at first 495 turns in its complete length, but was otherwise exactly similar to those wound on brass. The error was discovered, and was corrected, in the earliest part of the work, although the difference was too slight to effect the results appreciably.

The standardizing current $I_s$ was obtained from a single dry cell in series with one or more high resistance coils whose resistance $Z$ was precisely known. Usually $Z$ was the resistance of a standard megohm box. It was never less than one megohm. The electromotive free $\psi$ of the cell was determined almost every night observations were made by comparison with a standard Weston cell, itself repeatedly checked.

The angular velocity $\Omega$ in radians per second was always

$$\Omega = 2\pi \times 60$$

For $e/m$ the value $1.759 \times 10^7$ was adopted. Thus the formula (16—15) becomes

$$\rho e/m = \frac{4\pi \times 44}{2.540}\,\frac{\psi\,(\text{volts})}{Z\,(\text{ohms})} \times \frac{1.759 \times 10^7}{10 \times 2\pi \times 60} \times \frac{R}{S}\cdot\alpha \qquad (17\text{—}1)$$

When, as usually, $Z = 1$ megohm, this formula becomes

$$\rho\, e/m = 1.0157 \times \psi \times \alpha \times R/S^{6} \qquad (17\text{—}2)$$

[6] With $Z = 1$ megohm and $\psi$ approximately equal to 1.5 volt the quantity $R/S$ was always less than unity. See the example in § 18. In the case of a few rotors observations with the Moll relay in use, and horizontal drive, were made both with $R/S$ less than unity, and with $R/S$ greater than unity, $R/S$ in one group being approximately equal to $S/R$ in the other. This was done in order to reduce any error which might originate in lack of proportionality between deflections

The errors in all of the constants are entirely negligible.

18. EXAMPLE OF A SET OF OBSERVATIONS. DETERMINATION OF $\rho$, $\tau$, $P$, AND $M/R$. The procedure in making the observations is described in the next section. An example of one set of observations (with vertical drive) is given in Table 18—2. The symbols $RR$, $RL$, etc. used in this table are interpreted in the preceding table, viz. Table 18—1. The first letter refers to the rotor, the second to the calibrating coil. Thus the symbol $RR$ means the deflection produced when the rotor, looked at from above in the case of vertical drive, or from the west end in the case of horizontal drive, is driven clockwise and the solenoid is so connected that it produces a deflection in the same direction as that produced by the gyromagnetic effect in the rotor.

The quantity $R$ is the deflection due to the gyromagnetic effect being investigated, $S$ the calibrating deflection, $T$ the deflection due to the torsion (and other extraneous effects proportional to the speed and reversed with the direction of rotation), $M$* the deflection due to centrifugal and other mechanical effects, and $P$ the deflection due to the fact that when the zero is read just before the motor starts the flux through the secondary circuit has not its mean value for all position angles with the rotor at rest. The subscript $U$ or $D$ indicates that the rotor mark is up or down ($W$ or $E$ for horizontal drive). All deflections are positive when directed to the observer's right.

Each set is divided into two equal parts, one with the rotor mark up, the other with the mark down. Each of these parts is divided into two subgroups, 1 and 2, for which the initial position angles of the rotor are set 180° apart (so that $P_1 = -P_2$).

We thus obtain Table 18—1, where $\overline{RR}$, $\overline{LL}$, etc. are the mean values of the deflections contained in the columns of any subgroup.

In each half set $\Sigma_1$ is the sum of all the mean deflections for the subgroup $P_1$, and $\Sigma_2$ the corresponding sum for the subgroup $P_2$.

$M$ and $P$ have always been calculated for each half set from the formulas given in Table 18—1. The quantities $R \pm T$ and $S$ have always been calculated for the half-groups separately, as indicated in Table 18—2, as an additional check on the work. Also, by substituting $(R + T)_U$ for $R_U$ in the formula for $\rho$, $\rho_u$, the apparent value of $\rho$ for mark up has been calculated; and similarly, by substituting $(R - T)_D$ for $R_D'$ in the formula, the *apparent* value $\rho_D$ has been calculated. The true value of $\rho$ is $\frac{1}{2}(\rho_U + \rho_D)$. The quantity $\frac{1}{2}(\rho_U - \rho_D) = \tau$ measures the error in $\rho$ which would have been made on account of torsion etc. except for the end-for-end reversal of

and fluxes. With one rotor, Heusler Alloy, but by no means certainly for this reason, a systematic difference did appear between the results of the two groups; but there was no certain difference with the others. The mean value of $\rho$ was always taken as correct. The best arrangement would doubtless have been such as always to make $R/S = 1$.

* The effects producing $M$, eliminated by reversal of the direction of rotation, are more complex than was supposed in the earlier work.

TABLE 18—1

INTERPRETATION OF EXPRESSIONS $RR$, $RL$, ETC.

| P | Mark Up (or $W$) | Mark Down (or $E$) |
|---|---|---|
| (1) | $RR = R + S + T + M_u + P_U$ | $RR = R + S - T + M_D + P_D$ |
| | $LL = -R - S - T + M_u + P_U$ | $LL = -R - S + T + M_D + P_D$ |
| | $RL = R - S + T + M_u + P_U$ | $RL = R - S - T + M_D + P_D$ |
| | $LR = -R + S - T + M_u + P_U$ | $LR = -R + S + T + M_D + P_D$ |
| (2) | $RR = R + S + T + M_u - P_U$ | $RR = R + S - T + M_D - P_D$ |
| | $LL = -R - S - T + M_u - P_U$ | $LL = -R - S + T + M_D - P_D$ |
| | $RL = R - S + T + M_u - P_U$ | $RL = R - S - T + M_D - P_D$ |
| | $LR = -R + S - T + M_u - P_U$ | $LR = -R + S + T + M_D - P_D$ |
| | $\overline{RR} - \overline{LL} = \overline{2S} + \overline{2(R+T)}$ | $\overline{RR} - \overline{LL} = \overline{2S} + \overline{2(R-T)}$ |
| | $\overline{LR} - \overline{RL} = \overline{2S} - \overline{2(R+T)}$ | $\overline{LR} - \overline{RL} = \overline{2S} - \overline{2(R-T)}$ |
| | $\Sigma_1 + \Sigma_2 = 8M_U \quad \Sigma_1 - \Sigma_2 = 8P_U$ | $\Sigma_1 + \Sigma_2 = 8M_D \quad \Sigma_1 - \Sigma_2 = 8P_D$ |

TABLE 18—2

EXAMPLE OF ONE SET OF OBSERVATIONS (ON COBALT I). VERTICAL DRIVE.

A. Observations

Deflections in cm.

*Mark Up*

| $P$ | $RL$ | $LR$ | $LL$ | $RR$ | $LL$ | $RR$ | $RL$ | $LR$ |
|---|---|---|---|---|---|---|---|---|
| | −10.96 | − 7.53 | −18.06 | − 0.27 | −17.96 | − 0.27 | −10.93 | − 7.60 |
| (1) | 10.95 | 7.66 | 17.97 | 0.26 | 18.04 | 0.33 | 10.78 | 7.56 |
| | 10.67 | 7.60 | 17.94 | 0.46 | 17.97 | 0.32 | 10.95 | 7.58 |
| | −10.02 | − 7.30 | −17.83 | − 0.24 | −17.68 | − 0.10 | −10.58 | − 7.37 |
| (2) | 10.63 | 7.23 | 17.68 | 0.20 | 17.76 | 0.07 | 10.64 | 7.53 |
| | 10.83 | 7.13 | 17.77 | 0.04 | 17.82 | 0.14 | 10.55 | 7.35 |

$(R + T) = 3.60$ $S_1 = 5.24$ $(R + T)_2 = 3.60$ $S_2 = 5.21$

*Mark Down*

| $P$ | $RL$ | $LR$ | $LL$ | $RR$ | $LL$ | $RR$ | $RL$ | $LR$ |
|---|---|---|---|---|---|---|---|---|
| | + 7.34 | +11.02 | + 0.50 | +17.66 | + 0.24 | +17.74 | + 7.37 | +10.97 |
| (1) | 7.27 | 10.93 | 0.16 | 17.77 | 0.29 | 17.76 | 7.16 | 10.76 |
| | 7.19 | 10.76 | 0.17 | 17.86 | 0.17 | 17.74 | 7.22 | 10.85 |
| | + 7.20 | +10.66 | 0.00 | +17.46 | + 0.03 | +17.36 | + 7.04 | +10.68 |
| (2) | 6.87 | 10.53 | + 0.06 | 17.36 | 0.03 | 17.55 | 6.77 | 10.53 |
| | 6.90 | 10.63 | 0.03 | 17.47 | 0.06 | 17.47 | 6.85 | 10.46 |

$(R - T)_1 = 3.47$ $S_1 = 5.28$ $(R - T)_2 = 3.46$ $S_2 = 5.28$

$\psi = 1.572$ volt $Z = 1$ megohm $[\alpha = 1.007]$

B. Results from Above Set of Observations

| $e/m \times \rho_U = e/m \times (\rho + \tau)$ | $e/m \times \rho_D = e/m \times (\rho - \tau)$ | $e/m \times \tau$ | $\rho\, e/m$ | $R_U$ | $R_D$ | $M_U/R_U$ | $M_D/R_D$ | $M/R$ | $P_U$ | $P_D$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.108 | 1.055 | +0.026 | 1.082 | +3.60 cm | +3.46 cm | −2.52 | +2.57 | −2.54 | −0.13 cm | +0.15 cm |

the rotor, or the end-for-end reversal of the drive when the axis of rotation was horizontal.

In Table 18—2, B, as in the corresponding parts of Table 20—1, below, $M/R$ is the mean of the values of $M/R$ and $M_D/R_D$, the sign of the second being reversed.

19. PROCEDURE IN MAKING THE PRINCIPAL OBSERVATIONS. All the observations were made on a regular time schedule. At the beginning of a particular minute on the clock, the rotor having been set at the predetermined position angle (1), and the galvanometer scale having been (slightly) shifted to zero, the switches were closed to produce the deflection $RL$ (See § 18). An automatic signalling device caused a neon lamp to glow and a telegraph sounder to click at the end of a fixed interval (usually about $10^s$ or $8^s$, according to the galvanometer circuits in use) after the switches were closed. The galvanometer was read at this instant, and the switches were then immediately opened.

From the beginning of the next successive minutes on the clock the procedure was repeated, except that the switches were set for the deflection $LR$, $LL$, etc., until the eight observations of the first line of Table 18–2 were completed. Then

the observations were repeated, in the same order, as indicated in the second and third lines.[7] The galvanometer drift was adjusted, if necessary, at the beginning of each line.

The whole set of 24 observations was then repeated, except that the rotor was set to start from a position angle (2) differing 180° from the previous angle (1). In the case of vertical drive, or in the case of horizontal drive with a single motor in use, the whole procedure was then repeated, but with the rotor reversed in its bearings.

When two motors were in use, drive by west or left motor and drive by east or right motor were interchanged instead of reversing the rotor. The next night, however, the complete process was repeated with the rotor reversed; but if the left (or right) motor was used first the night before, the right (or left) motor was now used first.

In many cases the time interval between successive observations was $2^m$ instead of $1^m$; and in some cases it was $\frac{1}{2}^m$. In many cases the interval between the observation ending one line and that beginning the next was twice the interval between successive observations.

In nearly all cases the electromotive force of the cell furnishing the standardizing current was measured for each set of observations. In many cases the residual magnetic moment of the rotor was measured, often both at the beginning and at the end of the set.

20. The principal results obtained in this investigation. These are contained in Table 20—1.[8] The first column which requires comment is Col. 3. In connection with this column it should be said that in general all the different groups for a given rotor were not made consecutively, but were distributed (when there was a sufficient number of them) through the work so as to check, if possible, systematic errors which might be different for different conditions.

In Col. 4 the numbers in parentheses are the numbers of sets made in Part I, when the Inca coils were in use. Also, except those at the beginning and end of Part I, the observations were made with the Moll relay in use. These observations in Part I are probably inferior to the others. Also they give, on the whole, somewhat smaller values for $\rho$. But they have been given equal weight with the others in making the calculations.

Col. 5 gives the mean values of $\rho e/m$ for all the sets, for both horizontal drive and vertical drive, in case both were used. Col. 6 gives the mean for both methods of drive, equal weights being attached to the two. Col. 7 gives the mean value when the observations in each group are weighted proportionally to their numbers. There is little difference between the two.

Col. 8 gives the value of $\tau$ calculated on the assumption that the deflection $T$ reverses with the reversal of the rotor's axis, and also with the change of drive from motor east to motor west, or vice versa, when two motors are used. The quantity does not represent an error, but represents the error which would be made except for the reversals introduced to eliminate it.

Col. 9 gives the quantity $M/R$, the mean of the values of $M_L/R_L$ and $M_R/R_R$ (or $M_U/R_U$ and $M_D/R_D$), the sign of the second being reversed.

Col. 10 gives the mean value, without regard to sign, of the position angle deflection $P$ to the gyromagnetic deflection $R$.

Col. 11 gives the quantity $L \equiv$ one-half the excess of $\rho\, e/m$ determined by observations with the left motor alone (for both directions of the rotor mark) over the corresponding value for the right motor. It does not represent an error, but represents the error which would result (in case of horizontal drive) if both motors were not used.

Col. 12 gives the quantity $\alpha$ which, for any rotor, turns out to be identical for the different coil arrangements used in the two methods of drive.

[7] Instead of three lines of observations, four lines were obtained in a number of sets—half with the galvanometer switch in one direction, half with the switch reversed.

[8] In addition to the results given in this table, two *partial* sets were obtained with the rotor Nickel IV, which was peculiarly susceptible to the effects of mechanical strain, and a number of sets with the rotor Cobalt II, which had worked quite satisfactorily in investigation I, but which showed great discrepancies in the present work. These results have been rejected pending further investigation of the sources of trouble. The mean values of $\rho\, e/m$ for both substances would have been increased by the inclusion of the results mentioned.

**TABLE 20—1**

**Principal Results Obtained in This Investigation**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rotor | Drive | No. Groups | No. Sets | $\rho\, e/m$ | *I* Mean $\rho\, e/m$ | *II* Mean $\rho\, e/m$ | $\tau$ | $M/R$ | $\|P/R\|$ | $L \equiv \frac{1}{2}(\rho_L - \rho_R)\, e/m$ | $\alpha$ |
| Electrolytic Iron III | Horiz. | 8 | 25 (25) | 1.020±0.017 | [1.020±0.017] | [1.020±0.017] | +0.005 ±0.006 | −0.39 ±0.002 | 1.4 | −0.004±0.010 | 1.012 |
| Electrolytic Iron IV | Horiz.<br>Vert. | 2<br>1 | 8<br>4 | 1.034±0.004<br>1.024±0.024 | 1.029±0.005 | 1.031±0.011 | +0.014 ±0.010<br>−0.014 ±0.018 | −0.15 ±0.03<br>−0.04 ±0.04 | 0.2<br>0.04 | −0.016±0.014 | 1.030+ |
| Norway Iron | Horiz. | 1 | 4 (4) | 1.028±0.011 | [1.028±0.011] | [1.028±0.011] | +0.054 ±0.021 | +0.22 | 0.3 | | 1.015+ |
| Steel III | Horiz.<br>Vert. | 1<br>1 | 4 (4)<br>6 | 1.045±0.009<br>1.035±0.007 | 1.040±0.005 | 1.039±0.008 | +0.020[±0.010]<br>+0.002 ±0.007 | −0.37[±0.09]<br>−0.36[±0.01] | 0.07<br>0.03 | | 1.006 |
| Nickel I | Horiz.<br>Vert. | 6<br>1 | 24 (12)<br>1 | 1.050±0.030<br>1.067 | 1.052±0.008 | 1.052±0.030 | +0.005 ±0.004<br>−0.011 | −2.29 ±0.011<br>−2.45 ±0.05 | 0.30<br>0.25 | +0.002±0.005 | 1.010 |
| Cobalt I | Horiz.<br>Vert. | 2<br>2 | 8<br>8 | 1.070±0.007<br>1.074±0.004 | 1.072±0.002 | 1.072±0.006 | 0.000 ±0.003<br>+0.027 ±0.004 | −3.64 ±0.00<br>−2.71 ±0.11 | 0.12<br>0.03 | −0.015±0.002 | 1.007+ |
| Hopkinson's Iron—Nickel Alloy | Horiz.<br>Vert. | 2<br>2 | 8 (8)<br>11 | 1.018±0.012<br>1.019±0.019 | 1.018±0.000 | 1.019±0.016 | −0.004 ±0.010<br>0.000 ±0.003 | +1.89 ±0.03<br>+1.60 ±0.26 | 0.63<br>0.04 | +0.002±0.006 | 1.049 |
| Permalloy | Horiz.<br>Vert. | 2<br>1 | 11(11)<br>6 | 1.037±0.001<br>1.053±0.006 | 1.045±0.008 | 1.043±0.003 | +0.004 ±0.001<br>0.000[±0.001] | +0.14 ±0.04<br>+1.22[±0.001] | 0.72<br>0.01 | +0.001±0.002 | 0.991 |
| Cobalt-Iron | Horiz.<br>Vert. | 2<br>1 | 16 (16)<br>6 | 1.028±0.011<br>1.030±0.004 | 1.029±0.001 | 1.029±0.009 | +0.001 ±0.009<br>0.000[±0.003] | +0.46 ±0.00<br>+0.60[±0.04] | 0.7<br>0.01 | +0.009±0.001 | 0.994 |
| Cobalt-Nickel | Horiz.<br>Vert. | 3<br>1 | 14 (6)<br>5 | 1.085±0.015<br>1.054±0.006 | 1.077±0.015 | 1.077±0.013 | +0.014 ±0.006<br>+0.002 ±0.007 | +0.11 ±0.16<br>−0.55[±0.00] | 0.13<br>0.02 | −0.006±0.004 | 1.022 |
| Heusler Alloy (Cu, Al, Mn) | Horiz.<br>Vert. | 3<br>2 | 12 (12)<br>9 | 0.976±0.006<br>1.006±0.006 | 0.991±0.015 | 0.989±0.006 | +0.001±0.002<br>−0.004±0.001+ | +0.07 ±0.01<br>+0.05 ±0.01 | 1.3<br>0.05 | 0.000±0.002 | 1.052 |
| | | | | | | | | | | Mean $L \equiv \frac{1}{2}(\rho_L - \rho_R)\, e/m = -0.003 \pm 0.006$ for above 9 rotors. | |

21. Comparison of the results obtained in this investigation with those obtained in the other investigations in the author's laboratories. The results obtained in this investigation (III) are compared with those obtained in I and II in Table 21—1. For this purpose all the soft iron rotors are grouped together in each case, likewise the cold-rolled steel rotors, and the cobalt and copper-cobalt rotors.

The most precise values are doubtless those obtained in II. The gyromagnetic ratios for soft iron and permalloy obtained in it are probably current to one-half per cent or less; the others to one per cent, or about one per cent.

tained the first results on cobalt and nickel, and showed that the gyromagnetic ratios of these materials are nearly the same as that of iron, contained a (later detected) systematic error which made them certainly too large.

In the chart the abscissae, except for two substances, give the values of the gyromagnetic ratios from the author's investigation II, while the ordinates give the other results. In case of exact agreement with II any point would lie on the oblique straight line. The exceptional substances referred to are Heusler alloy and the oxides of iron, including magnetite. Heusler alloy was not studied (precisely) in II, so that

TABLE 21—1

Gyromagnetic Ratios of Ferromagnetic Substances. Values of $\rho\, e/m$ from the Three Extensive Investigations in the Author's Laboratories

($e/m$ = 1.759 e.m.u.)

| Material investigated | | Iron | Steel | Nickel | Hipernik | Hopk. Al. | Permalloy | Cobalt | Cobalt-Iron | Cobalt-Nickel | Heusler Alloy |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Investigation | Effect | | | | | | | | | | |
| I | Barnett | 1.049 | 1.047 | 1.031 | — | 1.016 | 1.054 | 1.070 | 1.067 | 1.068 | 1.011 |
| II | Einstein-de Haas | 1.032 | 1.038 | 1.051 | 1.051 | 1.023 | 1.046 | 1.085 | 1.025 | 1.076 | — |
| III | Barnett | 1.026 | 1.039 | 1.052 | — | 1.019 | 1.043 | 1.072 | 1.029 | 1.077 | 0.989 |
| Mean I & III | Barnett | 1.038 ±0.010 | 1.043 ±0.004 | 1.042 ±0.010 | — | 1.018 ±0.002 | 1.048 ±0.006 | 1.071 ±0.001 | 1.048 ±0.019 | 1.072 ±0.004 | 1.000 ±0.011 |
| Mean of II and Mean I & III | Both | 1.035 ±0.003 | 1.040 ±0.002 | 1.046 ±0.004 | | 1.017 ±0.001 | 1.047 ±0.001 | 1.078 ±0.007 | 1.036 ±0.012 | 1.074 ±0.002 | |

For the materials studied in all three investigations the mean values of $\rho\, e/m$ are as follows: (I) 1.050, (II) 1.047, (III) 1.045. The mean for (I) and (III) agrees exactly with the mean for (II). For the individual materials the means between I and III are given in line 6 of the table. The means between the values in II and those in line 6 are given in line 7. It is probable, however, that the values of III are more nearly correct than the means between I and III.

22. Comparison between results obtained in the author's laboratories and those obtained elsewhere. This comparison is made graphically in Fig. 22—1. The chart does not include the author's first observations by a magnetometer method (1917) on cobalt, nickel and iron, which, while useful in that they con-

the values obtained in I and III are placed as ordinates against their mean as abscissa. Magnetite and the other oxides of iron, not investigated by the author, are given the same abscissa as soft iron because the recent and extensive work of Ray-Chaudhuri on these substances shows that their gyromagnetic ratios are identical with that of iron within the limits of the experimental errors. The results of Sucksmith and Bates on iron and nickel have been slightly altered on account of the adoption in their work of a slightly incorrect method of calibration.

The close agreement between the results of III and II is apparent, also the generally good agreement between the results of I and II. The mean for iron in I is too high because the exceptionally large value obtained for the gyromagnetic

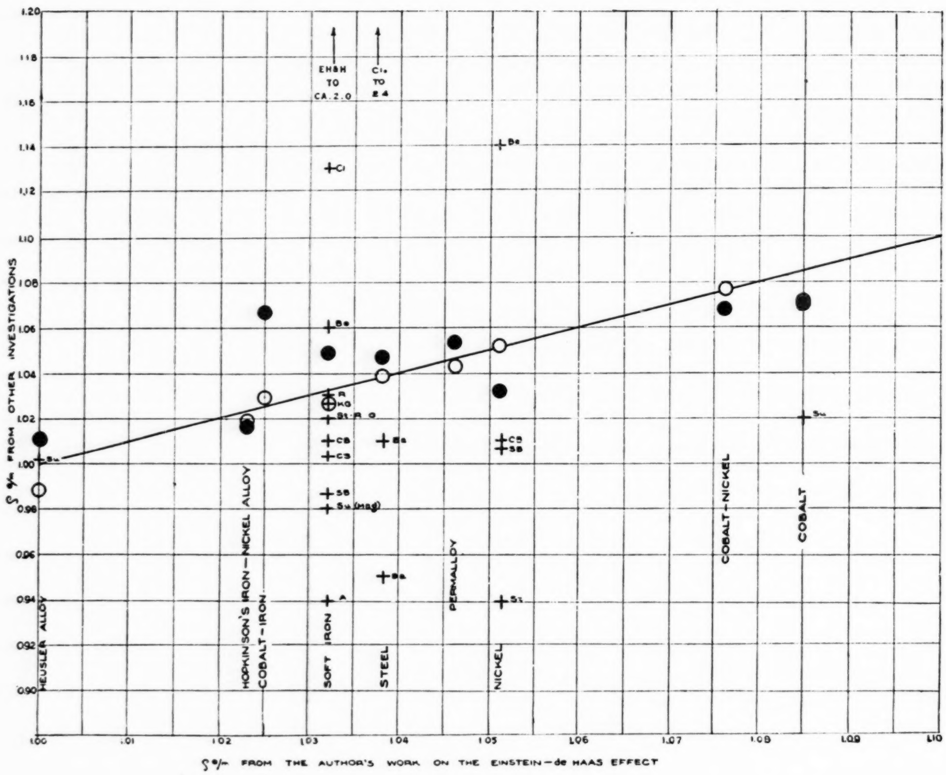


FIGURE 22–1. In this figure the empty circles refer to the present investigation (III); the full circles refer to the investigation I (Barnett & Barnett); the crosses followed by letters refer to other work as follows:

A stands for the work of Arvidsson (1919)
Ba stands for the earliest work of the author (1914 and 1915)
Be stands for the work of E. Beck (1919)
Cl stands for the work of Claassen (1922)
CB stands for the work of Chattock & Bates (1923)
CS stands for the work of Coeterier & Scherrer (1933–1935)
EH stands for the work of Einstein & de Haas (1915–1916)
G stands for the work of Galavics (1939)
H stands for later work of de Haas (1921)
KG stands for the work of Kikoin & Goobar (1938)
R stands for the work of Ray-Chaudhuri (1935)
Su stands for the work of Sucksmith (1925)
SB stands for the work of Sucksmith & Bates (1923)
St stands for the work of J. Q. Stewart (1918)

ratio of electrolytic iron was included. The value for cobalt-iron in I is likewise high. It seems probable that both these chief discrepancies, and some others which occur, are due largely to the not quite complete elimination of errors caused by mechanical strain.

All the most recent values for iron and iron oxides, viz. those of Ray-Chaudhuri, Kikoin and Goobar, and Galavics, are in closer agreement with my value of $\rho$ $e/m$ = 1.032 for iron than could be expected from their authors' estimated errors alone. For the somewhat older value $\rho$ $e/m$ = 1.00+ of Coeterier and Scherrer no claim of precision is made.[9] This is likewise true for the larger and, as I believe, more nearly correct value of Galavics.

[9] In the discussion at Strasbourg, Casimir, who was in Zürich while these experiments were being made, stated that the authors considered the possible error to be about 5 per cent.

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THE UNIVERSITY OF CALIFORNIA, LOS ANGELES CAMPUS
THE CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA
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